**Using Game Theory for Spectrum Sharing Specially in Cognitive Radios**

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Abstract

**The next generation of wireless networks is evolving towards networks of small, smart devices, which opportunistically share the spectrum with minimal coordination and infrastructure. The new emerging generation of networks consists of cognitive terminals, which intelligently and autonomously adapt to the channel environment to optimize their transmission parameters such as the band in which they transmit.** **In this paper, we address the problem of spectrum sharing between network operators and cognitive radios. Because of the dynamic nature of spectrum sharing, it is difficult to analyze and to provide sound spectrum management schemes. Several researchers rely on game theory that is an appropriate tool for modeling strategic interactions between rational decision makers (e.g., spectrum sharing in wireless networks). We present a selected set of works to highlight the usefulness of game theory in solving the main problems in this field.**

Keywords**:** cognitive radio, game theory, spectrum sharing, auction design, graph coloring.

# Introduction

Wireless communications rely on the frequency spectrum as a fundamental resource. As the number of wireless communication technologies and the number of wireless networks using them kept increasing, the regulation of the access to the available frequency spectrum, i.e., controlled spectrum sharing, has become unavoidable. A straightforward solution for the spectrum sharing problem is to let government agencies, such as the FCC, allocate communication frequencies to different wireless networks. This was first practiced, and basically still is, on a first-come first-served basis and then by auctions [1]. The allocated right, called the spectrum license, grants an exclusive usage of a given frequency band to a certain company for a given purpose. The main problem with the licensed spectrum is that the licenses are typically established for long periods of time. Recent performance studies [2, 3] have shown that this significantly affects efficiency.

About 30 years ago, government agencies realized that the available spectrum was scarce and reserved certain frequencies as unlicensed bands for common use. Unlicensed bands eliminated the lengthy process of spectrum licensing thus allowing companies to enter into the communication market quickly. Government agencies limited the transmission power of wireless devices in unlicensed bands (this limit can vary among technologies). Yet, unlicensed bands can be quickly saturated, which also means that, in contrast to licensed usage, the quality-of-service (QoS) is hardly guaranteed by these networks. Although unlicensed bands have improved the overall spectrum utilization, they still do not solve the inflexibility caused by the licensing process.

Cognitive radio [4, 5, 6] is an emerging technology that enables devices to determine which of the available frequencies are unused, and to use them even if they are licensed to others. One fundamental requirement of these devices is that they should not interfere with the communication of the primary users, who obtained the license for the given frequency band. Game and auction theory are useful tools to study the strategic behavior of network participants; on the other hand, graph coloring techniques can be used to assess the system optimum solution in many problems, such as the channel allocation problem. In Section II we provide a short introduction to these analytical tools.

Our goal in this paper is to present a selected set of contributions in this field and to provide a better understanding of the current research efforts in this field. We give a high-level overview of the schemes.

We can divide the spectrum sharing games into two main groups, according to the players of the games: unlicensed band wireless systems, and cognitive radios. In Section III, we address the problem of unlicensed spectrum sharing. Finally, the scenarios presented in Section IV are related to spectrum sharing by cognitive radios.

# Theoretical Background

Game theory, auction design, and graph coloring are the main tools for the analysis of the spectrum sharing schemes presented in this paper. In this section, using a practical example, we introduce the fundamental concepts of non-cooperative game theory, such as Nash equilibrium (NE) and Pareto-optimality. Then,

we examine the benefits of using auctions in spectrum assignment and spectrum sharing design. Finally, we briefly discuss graph coloring techniques.

1. **Game Theory**

Game theory [7, 8, 9] is a discipline for modeling situations in which decision makers have to make specific actions that have mutual, possibly conflicting, consequences. The basic elements of a game G are the players, the strategies and the payoffs and can be shown in strategic form by G = (P, S, U). P, S, and U are the set of all players, the joint set of the strategy spaces, and the set of payoff functions of all players, respectively. Considering a player *i*, *−i* represents all the players belonging to P except *i* himself, they are often designated as being the opponents of *i*. corresponds to the strategy space of player *i* and the set of chosen strategies constitutes a strategy profile *s* (e.g., *s* = {} for two players). The payoff is the difference of the benefit and the cost of player *i* given the strategy profile *s*.

We can write the best response of player *i* to an opponent’s strategy vector as follows:

**Definition**: The best response of player *i* to the profile of strategies is a strategy such that:

= (1)

***Nash Equilibrium***

If two strategies are mutually best responses to each other, then the players have no reason to deviate from the given strategy profile.

**Definition**: The pure strategy profile constitutes a Nash equilibrium if, for each player i,

 (2)

This means that in a NE, none of the users can unilaterally change his strategy to increase his payoff. Alternatively, a NE is a strategy profile comprised of mutual best responses of all the players. Hence, the system is stable. It is worth mentioning that Nash [8, 10, 11] proved that every finite strategic-form game has a NE. Once we have verified that a NE exists, we have to determine whether it is a unique equilibrium point. If the players have identified various Nash equilibriums, it still might be difficult for them to coordinate on which one to choose.

***Pareto-optimality and Price of Anarchy***

One method to assess the efficiency of the equilibrium point in a game is to compare the strategy profiles using the concept of Pareto-optimality. To introduce this concept, we first define Pareto-superiority.

**Definition**: The strategy profile *s* is Pareto-superior to the strategy profile if for any player i:

 (3)

With strict inequality for at least one player. In other words, the strategy profile *s* is Pareto-superior to the strategy profile , if the payoff of a player *i* can be increased by changing from to *s* without decreasing the payoff of other players.

Note that the players might need to change their strategies simultaneously to reach the Pareto-superior strategy profile *s*. Based on the concept of Pareto-superiority, we can identify the most efficient strategy profile or profiles.

**Definition**: The strategy profile is Pareto-optimal if there exists no other strategy profile that is Pareto-superior to .

In a Pareto-optimal strategy profile, one cannot increase the payoff of player *i* without decreasing the payoff of at least one other player. Thereby, using the concept of Pareto-optimality, we can eliminate poor Nash equilibria by selecting those with a Pareto-superior strategy profile. Finally, a metric to measure

the quality of a given NE is the Price of Anarchy (PoA). First defined in [12], the PoA is the ratio between the worst NE and the Pareto-optimal.

1. **Auction Design**

In economics, an auction is a method to determine the value of a commodity that has an undetermined or variable price. In progressive auctions for example, the players propose increasing bids for a good, and the highest bid wins the auction. For the first time in July 1994, the FCC allocated the commercial spectrum via competitive auctions instead of the previous best public use method. Auctions are a suitable way to assign the spectrum licenses because it is the player who values the most the spectrum who obtains it. Still, the process can be long and can lead to an overestimated price due to strong competition. Hence, the rules for designing and conducting spectrum auctions has evolved towards more efficient mechanisms inspired by principles of game theory.

In part **b** of Section IV, we will discuss an auction scheme that is simple to implement in wireless cognitive networks.

1. **Graph Coloring**

Graph coloring consists in assigning a color to the vertices of a bidirectional graph *G = (V,E)*, where *V* is a set of vertices and *E* a set of edges. The coloring is a mark that defines the category of the vertex. Marking each vertex of a graph with a finite set of *k* colors is equivalent to partitioning the vertices into *k* categories. In the following, graph coloring will refer to the coloring of the vertices of a graph.

A coloring that uses at most *k* colors is called a *k-coloring*. Channel allocation problems, for example, can be solved by graph coloring algorithms. Let us assume that interferences in a wireless network with two channels are modeled as a conflict graph where each vertex is a mobile user and nodes are connected if they interfere as shown in Figure 1. The conflict graph can be reduced to a colored conflict graph representation in which colors map to channels. To minimize interference, the same channel must not be assigned to adjacent nodes and the problem is solved by finding a proper k-coloring of the conflict graph. A coloring is proper if no two adjacent vertices are assigned the same color (e.g., Figure 2). As graph coloring algorithms are NP-Complete, optimal graph coloring solutions are obtained via approximations [13]. In graph coloring approximation, the idea is to prioritize the graph coloring process, instead of exhaustively testing all color assignments.



**Fig.1:** Example of graph coloring algorithm: Conflict Graph representation of a wireless network with two channels A and B. The edges are labeled to indicate interfering channels.



**Fig.2:** Resulting proper 2-Coloring of the Conflict Graph. Note that node 3 could not be assigned a color.

# Games in Unlicensed Bands

The unlicensed bands are the radio spectrum that can be freely used without obtaining a license. In 1986, the FCC provisioned for the first time unlicensed bands for Industry, Science, and Medicine (ISM) applications based on spread-spectrum technologies in the 915 MHz, 2.4 and 5.7 GHz spectrum bands. In the 90s, the FCC allocated additional unlicensed bands at 2, 5, and 59-64 GHz for wireless applications which require small coverage. Spectrum sharing in unlicensed bands suffers from two main problems: (i) devices accessing unlicensed band may experience severe interference as they do not have exclusive access to the spectrum, (ii) spectrum sharing in unlicensed bands may result in the *tragedy of the commons* [14] as there are no inherent incentives to efficiently use the radio band [15]. In the following we study these two problems in the context of spectrum sharing among heterogeneous wireless systems [16] and among WiFi operators [17].

1. **Spectrum Sharing among Heterogeneous Wireless Systems**

 We first consider the situation where heterogeneous wireless systems (e.g., Bluetooth and IEEE 802.11 WiFi) share the spectrum of an unlicensed band where each system behaves selfishly and tries to maximize its transmission rate. Etkin *et al*. model and study in [16] the resulting interaction among the systems as a non-cooperative game, and proposed various spectrum sharing rules and protocols to allow the wireless devices to share the bandwidth in a fair and efficient way.

***Game Model :***

Suppose that *M* wireless systems, each consisting of a single transmitter-receiver pair, share an unlicensed band of *W* Hz. Let , be the power spectral density of the transmitted signal in system *i*, *i= 1, . . . ,M*, where the total power for each system cannot exceed , i.e. :

 (4)

Each system decides on a power allocation in order to maximize its transmission rate as follows. Given the power allocations of all other systems, system *i* chooses a spectral density, that maximizes where is the maximal rate that system *i* can achieve. Etkin *et al*. show in [16] that frequency-flat allocation given by :

 (5)

is always a NE for the above game. The rate allocation under the frequency-flat allocation is generally not Pareto efficient [16], and the outcome of the game may lead to a poor overall system performance. But in particular, the strategy which uses a repeated game model can be used to select a Pareto efficient power allocation as the NE.

1. **Spectrum Sharing among WiFi Operators**

Now, we consider the situation where several WiFi operators share a common unlicensed band that is sub-divided into a fixed number of channels. Each WiFi operators owns several Access Points (AP) and has to decide on the channel that each AP uses. If two APs (of the same or two different WiFi operators) are within a (sufficiently) small distance of each other, then they will interfere if both are assigned the same channel. Therefore, in order to ensure an acceptable level of service to their mobile users, neighboring APs must be assigned different channels. Halldorsson *et al*. [17] model the above channel assignment problem as a game between WiFi operators where each operator decides on a channel assignment for its own APs in order to maximize the total number of mobile users that it can serve. The outcome of the game is evaluated by means of the Price of Anarchy (PoA). The PoA measures how far the outcome of the game is from a social optimal channel allocation (i.e., a channel allocation that maximizes the total number of mobile users that are served by APs).

***Game Model :***

Consider a set *V* of APs that are owned by several WiFi operators. Let be the distance between the two APs *u* and *v*, and let and be the transmission and sensing range of AP *u*. Note that they depend on the transmission power of AP *u*. Let *G = (V,E)* be the corresponding interference graph where there is an edge between AP *u* and *v* if they are located close enough. We say that AP *u* and *v* are neighbors if there is an edge in *G* between *u* and *v*. There are *k* channels available in the unlicensed band. Operators activate APs sequentially where the order with which APs are set up is given by an external process (i.e., operators do not decide when to activate an AP). Whenever an AP is set up, the corresponding operator must choose a channel that does not interfere with any of the previously set up APs. In addition, an operator is allowed to change the channel assignments of the APs that it controls as long as it does not cause any interference with APs of other operators. When an operators sets up an AP, it only has knowledge about the channel used by neighboring APs that have already been set up. It does not have any knowledge about the channels used by previously set up APs that are outside the interference region of the AP nor does it have any knowledge about the order with which APs are set up. The payoff that an operator receives for setting up an AP is equal to the expected number of mobile users that it can serve with the AP, where different APs can have different payoffs. The goal of each operator is to choose channels in order to maximize the overall payoff of its APs.

***Channel Allocation Results :***

For the above game, each NE corresponds to a maximal *k-colored* subset of the graph *G*. This allows using graph theory results to compute the price of anarchy for the above game. In particular, Halldorsson *et al*. derive in [17] the following results. For the general case where APs can have different transmission powers and different payoffs, the price of anarchy is potentially unbounded, i.e., PoA = ∞ (see Figure 3). The price of anarchy is also unbounded for the case where APs have different transmission powers, but all APs have the same payoff. If all APs have the same trans. power and the same payoff, then PoA is at most *5 + max(0, 1−5/k)* and at least *5*.



**Fig.3:** Network operators A and B with APs provide the Internet access to mobile users. The mobile users do not have Internet access because they are in the proximity of operator B, while operator A controls the channel. The PoA increases with the number of mobile users and is potentially infinite.

***Local Bargaining***

The above results show that if operators are forced to decide on a channel as soon as it is set up and are only allowed to reassign channels among the APs they control, the above game can lead to a poor coverage. An approach to improve performance is to allow operators to negotiate changes to the channel assignments of the APs that they control. Halldorsson *et al.* consider in [17] such an approach where operators can use channel bargaining to locally optimize their total payoff. In particular, they consider two bargaining schemes called *local 2-buyer-1-seller bargains* and *local 1-buyer-multiple-seller bargains*. Figure 4 illustrates the local 2-buyer-1-seller bargain.



**Fig.4:** Example of 2-buyer-1-seller bargain: (a) a1 controls the channel, but the sum of b1 and b2 payoffs (i.e., the number of mobile users) is greater than a1 payoff. (b) b1 and b2 bargain with a1 to acquire the channel and improve the system payoff.

***Game Results :***

Halldorsson *et al*. show in [17] that for the general case where APs have different transmission power and different payoffs, PoA is unbounded even if local bargains are allowed. For the case where all APs use the same transmission power and have the same payoff, PoA under local 2-buyer-1-seller bargains is at most *3+max(0, 1−3/k)* and at least *3*. For local 1-buyer-multiple-seller bargains PoA is at most *5+max(0, 1−5/k)* and at least *5* for the case where all APs use the same transmission power but have different payoffs. Halldorsson *et al*. also show that the above bargaining schemes will converge to a NE after a polynomial number of steps as a function of the number of APs, given that the payoffs are integers bounded by a polynomial in the number of APs. They also consider in [17] more general bargaining schemes. They prove that generally local bargaining may still lead to a poor performance unless the channel assignment of a large number of APs can be changed i.e. global bargaining at each bargaining step.

#  Cognitive Radio Games

Cognitive radios can detect whether a certain radio band or channel is currently used, as well as sense the amount of interference (interference temperature) within a given radio band or channel [15]. In addition, they are able to control the transmission powers and dynamic spectrum management with the help of *software defined radios* [15, 19]. These capabilities open the possibility of a flexible sharing of the wireless spectrum [19].

In this section, we focus on the scenarios where cognitive radios are used to efficiently share the available spectrum. We first consider the situation where a primary user (operator) acquires and owns a licensed radio band. If the primary user does not fully utilize this band, then it can be accessed by secondary users (cognitive radios), as long as they do not create any (or a sufficiently small) interference to the primary user. In particular, we consider the the following two cases: (1) secondary users can freely utilize the radio spectrum as long as they do not interfere with the primary user (Part **a**) and (2) the primary user sells access to the radio band through an auction mechanism (Part **b**). In Part **c**, we consider an OFDM network where several cognitive radios share the available OFDMA channels.

1. **Opportunistic Spectrum Sharing**

We first consider the situation where several primary users acquire their own radio band and where each radio band is further divided into several channels. We focus on an opportunistic spectrum sharing where secondary users (cognitive radios) are free to utilize channels as long as they do not interfere with the primary users [18, 19, 20]. Here, it assumed that secondary users will cooperate with each other to obtain a channel allocation with a maximum utilization subject to a given fairness criteria. Figure 5 provides an example of opportunistic spectrum sharing where the unused spectrum from a TV broadcast channel is utilized to provide WiFi connections to a residential community. Secondary user *2* cannot make use of channel *A* as it would interfere with primary user *X*. Secondary users *1* and *3* can emit on channel *A* as long as they control their transmit power not to interfere further than or respectively.



**Fig.5:** Secondary users 1 and 3 exploit channel A with their WiFi APs without interfering with the base station of the primary user.

***System Model :***

Consider the situation where *N* secondary users share *M* channels. Different channels allow different secondary users to transmit at different rates. Let be the throughput that user *n* can achieve on channel *m*, where *n = 1, . . . ,N* and *m = 1, . . . ,M*. The interference constraints are modeled by an interference graph which defines on which channels a given secondary user does not interfere with the primary user, and which secondary user can simultaneously transmit on a given channel *m* without causing interference among themselves. In addition, it is assumed that the maximum number of channels assigned to a secondary user cannot exceed a given threshold . For a given feasible channel allocation (i.e., a channel allocation that does not violate any interference constraints and any of the constraints on the maximum number of channels allocated to a secondary user), the network utilization is defined as the sum of the throughput over all secondary users.

***Results :***

Using the above model, Peng *et al.* formulate in [19] the channel allocation problem as a graph coloring problem where the goal is to maximize network utilization, or to maximize network utilization subject to a given fairness criteria such as max-min fairness and proportional fairness [21]. Finding such an optimal channel allocation is NP-hard and Peng *et al.* propose several heuristic graph coloring algorithms for each of the above performance objective. Peng *et al.* derive lower bounds for the performance of the centralized as well distributed implementation of these algorithms.

The algorithms proposed by Peng *et al.* in [19] require coordination and frequent information exchange among secondary users, and may impose substantial overhead on the network. As an alternative approach, Zheng and Cao in [21] propose a so called device-centric management scheme where each secondary user accesses channels based on simple rules that require only local information. The authors propose several allocation rules and derive lower bounds for their performance in terms of the poverty line (i.e., the minimum number of channels each secondary user is guaranteed to obtain), An important property of the proposed algorithms is that they reach a stable channel allocation in a finite number of iterations.

In [19], Cao and Zheng allow secondary users to be mobile. Instead of computing the channel allocation at each network topology change, secondary users negotiate channel allocations with their neighborhood via local bargaining. For the proposed local bargaining mechanism, a theoretical bound on the lower bounds for their performance in terms of the poverty line is provided.

1. **Auction Based Spectrum Sharing**

Next, we consider the situation where a primary user (operator or government agency) lets secondary users access its spectrum subject to a given power constraint, i.e., the total interference created by the secondary users at fixed measurements points has to be below a given threshold. For this situation, Huang *et al.* propose in [23] an auction-based spectrum sharing where secondary users submit bids. Based on these bids, the primary user decides on the transmission power allocated to each secondary user, as well as the cost per unit transmission power that secondary users are charged. The goal of each user is to submit bids in order to maximize its payoff minus cost, where the payoff is a function of the received signal-to-interference plus noise ratio.

***System Model :***

Consider *M* secondary users and let be the transmission power of user *i*, where *i = 1, . . . ,M*. The primary user then allocates transmission power to secondary users such that the total received power at a given measurement point is less than a threshold. The payoff function of secondary user *i* is a function of signal to noise and interference ratio (SINR) at secondary user *i*’s receiver given by , where is a user-dependent parameter.

***Auction Based Allocation :***

For the above situation, Huang *et al.* consider the problem where the primary user wants to allocate transmission power to secondary users in order to maximize the social welfare [23] subject to the interference constraint at the measurement point. Huang *et al.* propose the following auction based power allocation scheme. The primary user decides on a reserve power and announces a reserve bid and a price . After observing and , each secondary user *i* submits its bid . The primary user then allocates transmission power to secondary users so that the received power at the measurement point is proportional to the bids and charges each secondary user a price . The goal of each secondary user is to submit a bid such that the resulting power allocation maximizes its payoff minus cost. Assuming complete information, Huang *et al.* model this situation as a non-cooperative game, and prove that for > 0, there exists a threshold price > 0 such that a unique NE exists if ; there does not exist a NE if . For this game Pareto optimality and stability (NE) are conflicting, however an ε-Pareto optimal NE can be achieved. The ε-Pareto optimal allocation is the Pareto-optimal solution for the ε-system in which the total received power at measurement point is less than (1 − ε). The assumption that secondary users have complete information when deciding on their bids is unrealistic and Huang *et al.* propose an iterative bidding algorithm that requires each secondary user to have access only to his local information (i.e., its own payoff function and its local channel gains) and therefore can be implemented in a fully distributed manner. It is shown that this algorithm converges to the NE of the complete information game.

1. **Spectrum Sharing in OFDM Networks**

In this part we study spectrum sharing among cognitive radios in OFDMA networks [23]. OFDMA is a transmission technique which divides the available spectrum into sub-carriers and hence can support different QoS by assigning different number of sub-carriers to the users. OFDMA has recently used in the WiMAX (IEEE 802.16 Wireless MAN) uplink. In the following, we consider the situation where several cognitive radios, each having its own QoS constraint in terms of throughput, compete for access to the available sub-channels in an OFDMA network. Note that, there is not any spectrum manager (auctioneer) in this OFDMA network.

***Sustem Model :***

Consider an OFDMA network consisting of *L* sub-channels that are shared among *K* users (cognitive radios). Each user has a given QoS constraint in terms of throughput,(i.e., let Ri, i = 1, . . . ,K, be the required transmission rate of user i). Consider a given power allocation given by the matrix P where is the transmission power of user *i* on sub-channel *l*, *l=1,…, L*. The rate ,where is the SINR for user *i* on sub-channel *l* and *W* and *c* are known positive constants that depend on the system parameters. The goal of an OFDMA network provider is to minimize the overall transmission power subject to the given QoS constraints, and is a constraint on the maximal power at which a user can transmit. For the above optimization problem, we denote with Ω the set of feasible sub-channel rate allocations. In other words, Ω is the set of all sub-channel rate allocations , *i = 1,…,K* and *l = 1,…,L*, which satisfy the QoS constraints that is for *i=1,…,K* and for which there exists a feasible power allocation *P* such that, for *i = 1,…,K* and *l = 1,…,L.* Note that the set might be empty. Necessary conditions on the QoS constraints , *i = 1,…,K*, for to be non-empty are given in [23].

***Game Model :***

The above optimization problem is a generalized *knapsack* problem and finding an optimal power allocation is NP-hard [23]. Rather than relying on the network operator to decide on a power allocation, suppose that each user *i* is free to choose its own power allocation with the goal to satisfy the rate constraint with a minimal transmission power. The resulting interaction among users leads to the following non-cooperative game. Let be the sub-matrix indicating the power allocation over all users except user *i* and let be the power allocation of user *i*. Furthermore, let be the transmission rate of user *i* on sub-channel *l*, where is the SINR for user *i* on sub-channel *l* under the power allocation given by and as defined above. Given an allocation of all other users, user i chooses a power allocation to minimize its own transmission power subject to the QoS on its transmission rate by solving the following optimization problem,

 (6)

***Game Results :***

For the above non-cooperative game, Han *et al.* show in [23] that there always exists a NE if the set Ω is non-empty, i.e., if there exists at least one feasible power allocation that leads to the rate allocation that satisfies the QoS constraints , *i = 1,…,K.* However two difficulties arise in this context: (1) if the required rates , *i = 1,…,K*, are too large, then the set Ω might be empty and no NE exists, and (2) there might exist several NE, some of them with low system and individual performances. To overcome these difficulties, Han *et al.* consider the use of a virtual referee that is implemented by the network operator. Below we briefly describe the role of the virtual referee. Suppose that when the set Ω is empty and no NE exists, then a user *i* who is not able to satisfies the QoS constraint without violating the power constraint will decide on a power allocation as follows. Given a power allocation by all other users, user *i* chooses a power allocation that solves the following optimization problem,

 (7)

i.e. user *i* maximizes its throughput subject to the given power constraint. Han *et al.* show in [23] that there always exists a NE for the non-cooperative game based on the two optimization problems given by Equation (6) and (7), respectively.

However, the game may result in an undesirable NE with low system and individual performances. To improve system performance, the network operator implements a virtual referee that selectively restricts for some users the set of sub-channels that they are allowed to access. Han *et al.* provide in [23] the rules that the virtual referee uses to limit sub-channel access, one rule being that each user must have access to at least one sub-channel. Through numerical results, Han *et al.* illustrate in [23] that the resulting spectrum sharing mechanisms can significantly outperform two benchmark mechanisms where each sub-channels is allocated to at most one user (no sub-channel sharing) and where all users access all sub-channels (complete sub-channel sharing).

# Conclusions

In this paper, we have provided a detailed description of several research contributions in the area of spectrum sharing games. More specifically, we have focused on two kinds of players: network operators and cognitive radios. The reason of this choice is that we believe that the described scenarios will be among the most relevant ones in the coming years.

They provide insights on the possible consequences of greedy behavior in the either. As we have explained, most of the cases that we have described are amenable to modeling by game theory. In this way, it is possible to predict potential outcomes of the observed conflicting situations. But as we have mentioned, in order to make the problem tractable, all authors have made relatively drastic assumptions, notably in terms of information possessed by the players. Much research is still needed in this field, in particular to better capture the perception that each of the players has of the context in which it operates.

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